16th International Conference on
AEROSPACE SCIENCES & AVIATION TECHNOLOGY,
ASAT - 16 – May 26 - 28, 2015, E-Mail: asat@mtc.edu.eg
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Carrier Frequency Offset (CFO) Estimation Methods, A Comparative Study

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Abstract*:* Estimation of Carrier Frequency Offset (CFO) is the fundamental problem of OFDM systems. CFO can be estimated using either time domain or frequency domain methods. A comparative study among different methods of carrier frequency offset estimation in OFDM systems is presented. We consider two techniques in time domain, namely; the Cyclic Prefix (CP) method and the Training Sequence Method. In addition two techniques in frequency domain are considered, namely; Training Symbol method and Pilot Method. Mean square Error (MSE) is the comparison criteria used in the study. Simulation results show that the CFO estimation methods in frequency domain have better performance than CFO estimation methods in time domain, in terms of mean square error (MSE). It is shown that, the pilot tone method outperforms the cyclic prefix method by 7dB and the training sequence with two identical blocks by 10dB. The pilot tone method outperforms the symbol based method by approximately 2dB. The comparison results are considered at MSE of 10^{-4} and normalized CFO of 0.2.

Keywords: Carrier Frequency Offset (CFO), Non-Data-Aided (NDA), Data-Aided (DA), Cyclic Prefix (CP) and training symbols, Mean Square Error (MSE).

I. Introduction:

The common feature of the fourth generation (4G) of wireless communications technologies will be the convergence of different wireless networks with multimedia services such as speech, audio, video, image, Internet services, and data at high data rates and with high mobility, high capacity and high QoS. One of the most important technique is Orthogonal Frequency Division Multiplexing (OFDM), which fulfill these requirements. OFDM has recently gained a lot of attention and is a potential candidate for 4G wireless systems due to its bandwidth efficiency, high data rate transmission capability in a stationary and vehicular conditions and its robustness against multipath fading channels effects. OFDM is being used in a number of wired and wireless, voice and data applications due to its flexible system architecture. OFDM has been adopted by several technologies such as Asymmetric Digital Subscriber Line (ADSL) services, IEEE 802.11a/g, IEEE 802.16a, Digital Audio Broadcast (DAB), and digital terrestrial television broadcast, LTE, UMTS , WiMAX [1] , DVD in Europe, ISDB in Japan 4G, IEEE 802.11n, and IEEE 802.20 [2].

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OFDM is very sensitive to time and frequency synchronization. The synchronization problem consists of two major parts: carrier frequency offset (CFO) and symbol time offset (STO) [3]. This OFDM synchronization at the receiver is one important step that must be performed. Synchronization of an OFDM signal requires finding the symbol timing and carrier frequency offset. Finding the symbol timing for OFDM means finding an estimate of when the symbol starts. There are two main causes of CFO. The first is a frequency mismatch between the local oscillators at the transmitter and receiver, which results in residual CFO at the receiver after the down-conversion process. The second cause is the Doppler shift, which is a result of the relative motion between the transmitter and receiver present in mobile environments [4, 5]. The normalized CFO can be divided into two parts; integral CFO (IFO) ε_i and fractional. CFO (FFO) ε_f . IFO produces a cyclic shift by ε_i in receiver side to corresponding subcarrier. IFO does not destroy orthogonality among the subcarrier frequency components. However, FFO destroys the orthogonality between the sub-carriers [5].

CFO estimation is required to maintain or preserve the orthogonality properties of the subcarriers, because CFO can lead to the Inter Carrier interference (ICI)[6][7]. ICI means that, a subcarrier frequency component to be affected by other subcarrier frequency components [8] and degrade the OFDM system performance [9, 10]. There are several techniques to estimate and compensate CFO using time-domain or frequency-domain approaches, sometimes-called pre-FFT and post-FFT synchronization, respectively. Pre-FFT synchronization performs the estimation of CFO before OFDM demodulation (FFT processing). The pre-FFT approach provides fast synchronization and requires less computing power due to the fact that no FFT processing is needed. Pre-FFT synchronization can be classified into two categories: non-data-aided (NDA) and data-aided (DA) [7]. NDA methods exploit similarities between the cyclic prefix (CP) part and the corresponding data part of a received OFDM symbol to estimate fractional CFO [11]-[14]. This can be done by correlating the CP and the corresponding OFDM symbol to estimate both timing and frequency offsets. NDA methods that use the CP can only estimate CFO in the range of ±0.5 subcarrier spacing [11]. DA methods exploit a known sequence of OFDM training symbols inserted at the start of every OFDM packet (widely used in 802.11 WLAN [13]) to estimate fractional CFO. The downside of DA pre-FFT synchronization is reduced transmission efficiency due to the insertion of the training symbols. However, this method provides better results and a wider CFO estimation range than the NDA algorithms hence, DA Methods estimate CFO in the range of ±1 subcarrier spacing [15].

Post-FFT synchronization methods usually perform the estimation of the remaining integer CFO left by pre-FFT frequency synchronization. Integer CFO can be estimated by correlating the received pilot subcarriers with a shifted version of the known pilot sub-carriers [24]. Depending on spacing between pilot sub-carriers, this approach can estimate CFO range up to several multiple integers of sub-carrier spacing. Therefore, it is necessary to estimate the CFO, which explains distortion in the transmitted symbols and compensated using some estimation techniques in receiver. Both time-domain and frequency-domain synchronization play important roles in correcting carrier frequency offset in OFDM systems.

The rest of this paper is organized as follows. Two time-domain estimation methods are presented in section II. Two frequency-domain estimation methods are presented in section III. Simulation results are given in Section IV. Finally, conclusions are drawn in Section V.

II. Time-Domain Estimation Methods for CFO

Estimation techniques in the time domain (TD) are based on introducing a cyclic prefix, or training sequence, and the use of correlation [11]-[14]. Each of these techniques are discussed as follows.

A. CFO Estimation Method Using Cyclic Prefix (CP)

Cyclic prefix (CP) is a portion of an OFDM symbol used to absorb inter-symbol interference (ISI) caused by any transmission channel time dispersion and it can be used in CFO estimation. Figure (1) shows OFDM Symbol with CP. CP based estimation method exploits CP to estimate the CFO in time domain. Considering the channel effect is minimal and can be neglected, then, the *l*<u>th</u> OFDM symbol affected by CFO can be written as

$$y_l(n) = x_l(n)e^{\frac{j2\pi\varepsilon n}{N}} \tag{1}$$

Replacing *n* by (*n*+*N*) in equation (1) the corresponding CP in the OFDM symbol can be written as

$$y_l(n+N) = x_l(n+N)e^{\frac{j2\pi\varepsilon\,(n+N)}{N}} \tag{2}$$

$$y_l(n+N) = x_l(n)e^{\left(\frac{j2\pi\varepsilon n}{N}+j2\pi\varepsilon\right)} \tag{3}$$

By comparing equation (1) and (3), we can find that the phase difference between CP and the OFDM symbol is 2πε. Therefore, the amount of CFO can be found from the argument of the multiplication of OFDM symbol by the conjugate of its CP:

$$\hat{\varepsilon} = \frac{1}{2\pi}\, arg\{y_l^*(n)y_l(n+N\,)\}\,, \mathrm{n} = -1, -2, \dots, -N_g \tag{4}$$

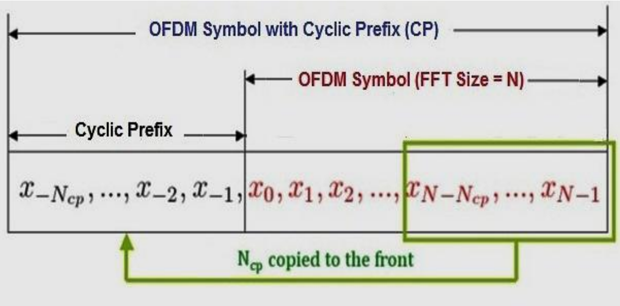

Fig. 1. OFDM Symbol with Cyclic Prefix

In order to reduce the noise effect, its average can be taken over the samples in a CP interval as:

$$\hat{\varepsilon} = \frac{1}{2\pi}\, arg\left\{\sum_{n=N_g}^{-1} y_l^*(n)y_l(n+N)\right\}, \mathrm{n} = -1, -2, \dots, -N_g \tag{5}$$

Since the argument operator arg( ) is performed by using $tan^{-1}(\,)$, the range of CFO Estimation in equation (5) is [−π,+π]/2π = [-0.5, +0.5] so that $|\hat{\varepsilon}| \leq 0.5$. Therefore, CP results CFO estimation in the range, $|\hat{\varepsilon}| \leq 0.5$. Hence, this technique is useful for the estimation of Fractional CFO (FFO). CFO estimation technique using CP does not estimate the integer offset [4]. To overcome this drawback, the training sequence technique is used to estimate CFO. This is helpful in increasing the range of the CFO estimation [16]-[22].

B. CFO Estimation Method Using Training Sequence

It has been shown that the CFO estimation technique using CP can estimate the CFO only within the range ($|\hat{\varepsilon}| \leq 0.5$). Since CFO can be large at the initial synchronization stage, we may need estimation techniques that can cover a wider CFO range. The range of CFO estimation can be increased by reducing the distance between two blocks of samples for correlation. This is made possible by using training symbols that are repetitive with some shorter period. Let *D* be an integer that represents the ratio of the OFDM symbol length to the length of a repetitive pattern as shown in Figure (2).

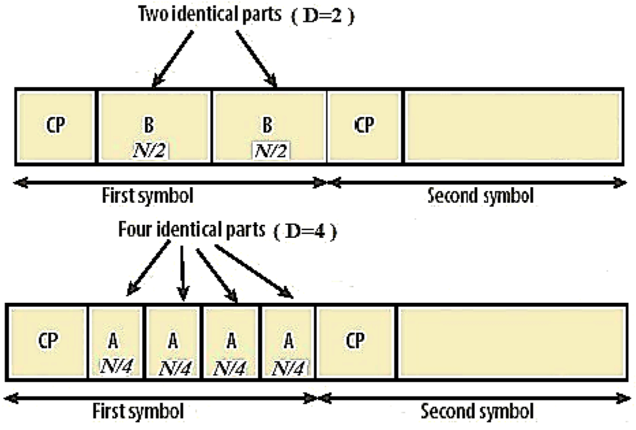

Fig. 2. Training Sequence in OFDM Symbol

Let a transmitter sends the training symbols with *D* repetitive patterns in the time domain, which can be generated by taking the IFFT of a comb-type signal in the frequency domain given as

$$X_l(k) = \begin{cases} A_m, & if\ k = D.i\,, i = 0,1,2,\dots,\left(\dfrac{N}{D}-1\right) \\ 0 & otherwise \end{cases} \tag{6}$$

Where A_m represents an Mary symbol and *N/D* is an integer. As $x_l(n)$ and $x_l(n+N/D)$ are identical, then, ($y_l^*(n)y_l\left(n+\frac{N}{D}\right) = |y_l(n)|^2\, e^{j\pi\varepsilon}$). A receiver can make CFO, estimation as follows

$$\hat{\varepsilon} = \frac{D}{2\pi}\, arg\left\{\sum_{n=0}^{N/D-1} y_l^*(n)y_l(n+N/D)\right\} \tag{7}$$

The CFO estimation range covered by this technique is ($|\hat{\varepsilon}| \leq D/2$), which becomes wider as *D* increases. Increasing in estimation range is obtained at the sacrifice of mean square error (MSE) performance. Hence, there is a trade-off relationship between the MSE performance and estimation range of CFO is clearly shown. This is due to reduction in the correlation samples by a factor of *1/D*. The solution to this problem is to calculate average of all the estimates over repetitive short periods as in equation (8).

$$\hat{\varepsilon} = \frac{D}{2\pi}\, arg\left\{\sum_{m=0}^{D-2}\sum_{n=0}^{N/D-1} y_l^*(n+m\,N/D)y_l(n+(m+1)N/D\,)\right\} \tag{8}$$

III. Frequency-Domain Estimation Methods for CFO

Frequency Domain (FD) CFO Estimation techniques are applied under assumption that perfect time synchronization is achieved. Furthermore, the FD techniques are based on transmitting two identical symbols or pilot tone (pilot insertion) [23], [24].

A. CFO Estimation Technique Using Training Symbol Method

Moose [23] considered the effects of frequency offset on the performance of OFDM systems. He has proposed the maximum-likelihood (ML) CFO estimation method based on two consecutive and identical training symbols. The same data frame is repeated and the phase value of the each carrier between consecutive symbols are compared as shown in Figure (3). The offset is determined by maximum likelihood estimation algorithm (MLE).

Moose's method can be explained as follows. An OFDM signal at the receiver, in the absence of noise, after repeating the same data frame is given by:

$$r_n = (1/N)\left[\sum_{k=-K}^{K} X_k H_k e^{2\pi jn(k+\varepsilon)/N}\right] \quad , \mathrm{n} = ,1, \ldots ,2\mathrm{N}-1 \qquad (9)$$

In equation (9), X_k is the transmitted signal, H_k is the transfer function of the channel at the *k*th carrier, and ε is the frequency offset. In order to determine the value of frequency offset, ε, compare the two consecutive received data symbols at a given frequency.

The *k*th element of the *N* point DFT of the first N points of equation (9) is

$$R_{1k} = \sum_{n=0}^{N-1} r_n e^{-2\pi jnk/N} \qquad ; k = 0,1,2,\ldots,N-1 \qquad (10\,)$$

And the *k*th element of the DFT of the second half of the sequence is

$$R_{2k} = \sum_{n=N}^{2N-1} r_n e^{-2\pi jnk/N} = \sum_{n=0}^{N-1} r_{n+N} e^{-2\pi jnk/N} \; ; k = 0,1,2,\ldots,N-1 \qquad (11)$$

From equation (9) we can see

$$r_{n+N} = r_n e^{2\pi j\varepsilon} \rightarrow R_{2k} = R_{1k} e^{2\pi j\varepsilon} \qquad (12)$$

In the presence of AWGN assumed W_{1k} and W_{2k}, the signal at the receiver becomes;

$$Y_{1k} = R_{1k} + W_{1k}$$

$$Y_{2k} = R_{1k} e^{2\pi j\varepsilon} + W_{2k}\,, \qquad k = 0,1,2,\ldots,N-1 \qquad (13)$$

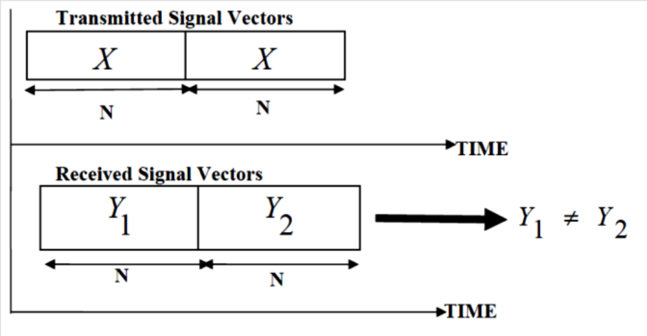

Fig. 3. Schematic of Moose Technique

Therefore, we can use a maximum likelihood approach to determine the relative frequency offset ε as

$$\hat{\varepsilon} = \frac{1}{2\pi} tan^{-1}\left[\frac{(\sum_{k=-K}^{K} Im[Y_{2k}Y_{1k}^{*}])}{(\sum_{k=-K}^{K} Re[Y_{2k}Y_{1k}^{*}])}\right] \tag{14}$$

Where $\hat{\varepsilon}$ is the maximum likelihood estimate of the relative frequency offset defined as $\varepsilon = N\Delta f/B$, where B is bandwidth, N is the number of subcarriers and Δf is the frequency offset in Hz. The estimation range of the Moose's method is equal to half ±0.5 sub-carrier spacing. Moose increased this range by using shorter training symbols, but that reduced the estimation accuracy.

B. CFO Estimation Method Using Pilot Method:

In the pilot tone method, we insert some pilot tones in the frequency domain and transmitted in every OFDM symbol, which uses in CFO estimation at the receiver after taking FFT. Figure (4) shows a structure of CFO using pilot tones. First, two OFDM symbols, $y_l(n)$ and $y_{l+D}(n)$, are saved in the memory after synchronization. Then, the signals are transformed into $\{Y_l(k)\}_{k=0}^{N-1}$ and $\{Y_{l+D}(k)\}_{k=0}^{N-1}$ via FFT, from which pilot tones are extracted. After estimating CFO from pilot tones in the frequency domain, the signal is compensated with the estimated CFO in the time domain. In this process, two different estimation modes for CFO estimation are implemented: acquisition and tracking modes. In the acquisition mode, a large range of CFO including an integer CFO is estimated. In the tracking mode, only fine CFO is estimated. The integer CFO is estimated by [24]

$$\hat{\varepsilon}_{acq} = \frac{1}{2\pi T_{sub}} \max\left\{\left|\sum_{j=0}^{L-1} Y_{l+D}(p(j),\varepsilon)\, Y_l^{*}(p(j),\varepsilon)\, X_{l+D}^{*}\big(p(j)\big)\, X_l\big(p(j)\big)\right|\right\} \tag{15}$$

where L, $p(j)$, and $X_l\big(p(j)\big)$ denote the number of pilot tones, the location of the jth pilot tone, and the pilot tone located at $p(j)$ in the frequency domain at the lth symbol period, respectively. Meanwhile, the fine CFO is estimated by [24]

$$\hat{\varepsilon}_f = \frac{1}{2\pi T_{sub}.D}\, arg\left\{\left|\sum_{j=0}^{L-1} Y_{l+D}\big(p(j),\hat{\varepsilon}_{acq}\big)\, Y_l^{*}\big(p(j),\hat{\varepsilon}_{acq}\big)\; X_{l+D}^{*}\big(p(j)\big)\, X_l\big(p(j)\big)\right|\right\} \tag{16}$$

In the acquisition mode, $\hat{\varepsilon}_{acq}$ and $\hat{\varepsilon}_f$ are estimated and then, the CFO is compensated by their sum. In the tracking mode, only $\hat{\varepsilon}_f$ is estimated and then compensated.

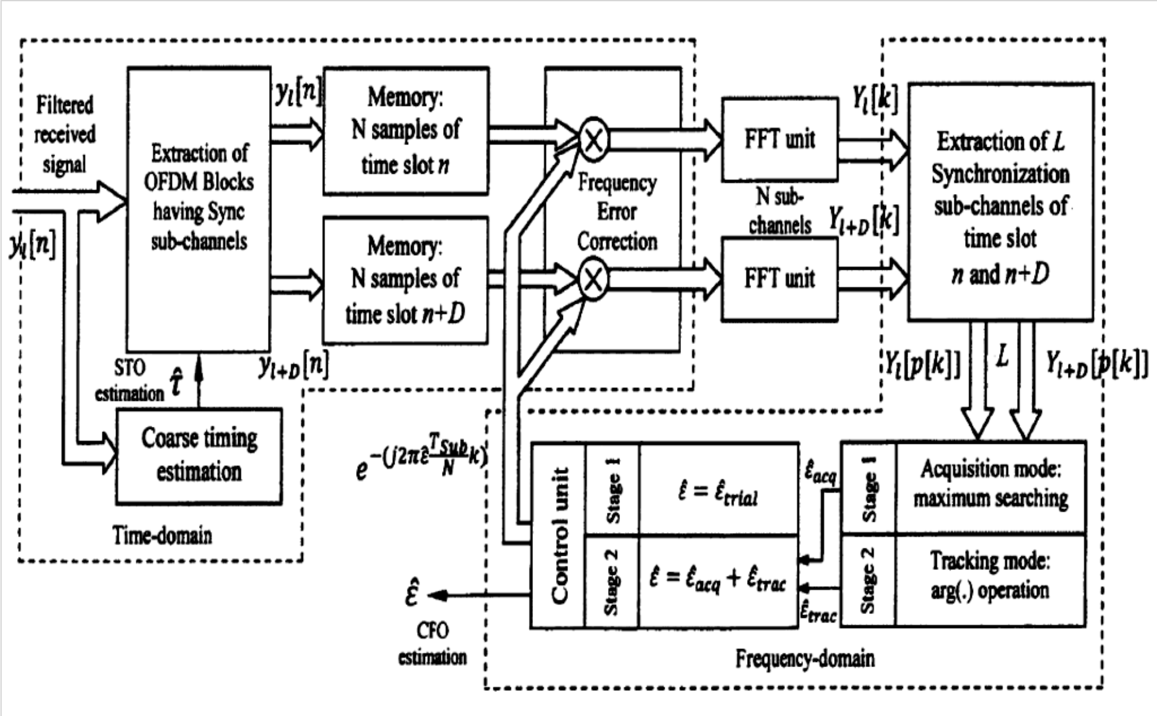

Fig. 4. CFO estimation scheme using pilot tones. [24]

IV. Simulation Results

In this section we show the comparison between the four previous CFO estimation techniques. The assumed simulation parameters are shown in Table (1). Figure (5) shows the MATLAB simulation results of CFO estimation using four different methods. The simulation results show that, at MSE of 10^{-4}, the pilot tone estimator outperforms the cyclic prefix estimator using 32 CP-length by 7dB and the training sequence estimator (D=2) by 10dB. The pilot tone estimator outperforms the symbol based estimator by approximately 2dB. The simulation results show the superiority of the pilot tone estimation method compared to other estimation methods.

Table 1. Assumed Simulation Parameters

No.	Parameter	Simulation Value
1	NFFT Size	128
2	Cyclic Prefix Length (Ng)	32
3	Modulation Scheme	QAM (QPSK)
4	Channel	AWGN
5	Normalized Frequency offset (CFO)	0.2
6	Number of Iterations	10000
7	Symbol Duration Length	3
8	Number of Bits per Symbol	2
9	Pilot Spacing (Nps)	4
10	Number of identical Parts (D)	2
11	Signal to Noise Ratio (SNR)	0-30 dB

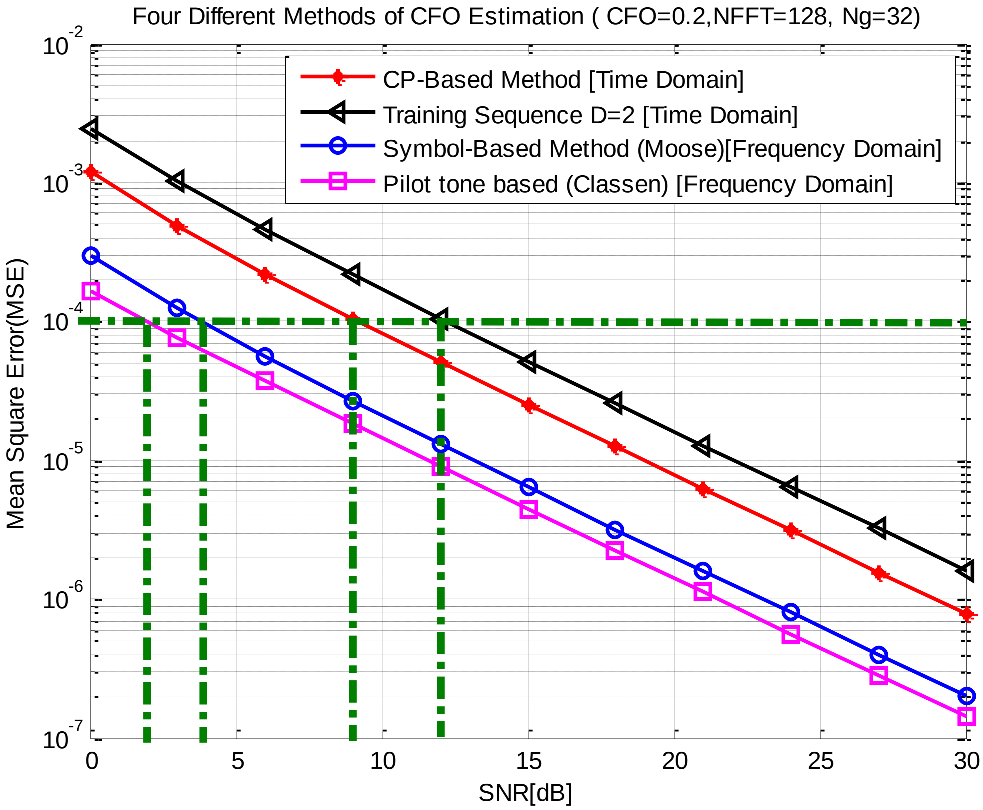

Fig. 5. Comparison between CFO Estimation Methods in AWGN Channel

V. Conclusion

In this paper, an overview and comparison of four different CFO estimation techniques in OFDM system in AWGN channel are considered. The mean square error (MSE) criteria is considered in the comparison. We examined two methods in time domain (pre-FFT) and two methods in frequency domain (post-FFT) that can estimate CFO. The CFO estimation methods in frequency domain using Pilot tone-based method and Symbol-based method have better performance than the estimation methods in time domain using CP-based method and training sequence method. The pilot tone method has the best performance.

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